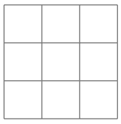
HÖJDPUNKTEN 2023

Högstadietävling, 10-11 of March 2023

Time: 3 hours
Tools: Only pen, eraser, compass and ruler
Full written solutions are required, in general no points are given for only writing the answer.

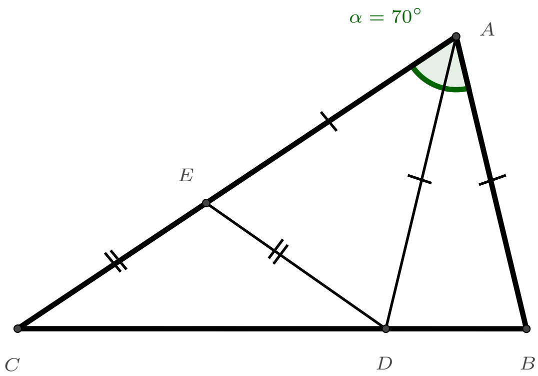
Problem 1. In the square grid below, the numbers $1-9$ should be placed such that each number appears exactly once. We want the sum of the numbers in each 2×2 subsquare to be the same, no matter which sub-square we choose (note that there are four such sub-squares). What is the smallest possible value of this sum?



Problem 2. In a pot there are balls of different colours. More than 61 percent of the balls and less than 65 percent of the balls are green. What is the smallest possible number of balls in the pot?

Problem 3. Kalle and Lisa are playing a game. First, Kalle picks 4 positive integers and tells Lisa which numbers he picked. Lisa then tries to create a number divisible by 6 using only the four numbers and the operations $+, -, \times$ and $\div$. She may do the calculations in any order (so she can use parentheses), but she must only use each number once. She wins if she succeeds, and Kalle wins otherwise. Can Lisa always win, no matter what numbers Kalle picks?

Problem 4. It is given that $\angle BAC = 70°$ in the triangle ABC. A point D is chosen on the side BC and a point E is chosen on the side AC, such that $|AB| = |AD| = |AE|$. It turns out that $|DE| = |EC|$. Determine all angles in the triangle ABC.



Problem 5. There are 100 people sitting in a circle. We know that some of them are liars who always lie, and some of them are truth tellers who always tell the truth, but we do not know who is what. Everyone in the circle says that they are sitting next to at least one liar. What is the smallest and largest possible number of liars in the circle?

Problem 6. For an integer x, we define $x^2, x^3, x^4, \ldots$ in the following way:

$$x^2 = x \cdot x$$

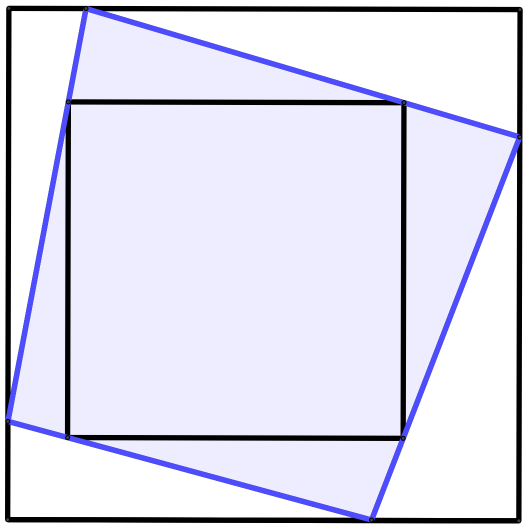
$$x^3 = x \cdot x \cdot x$$

$$x^4 = x \cdot x \cdot x \cdot x$$

$$\ldots$$

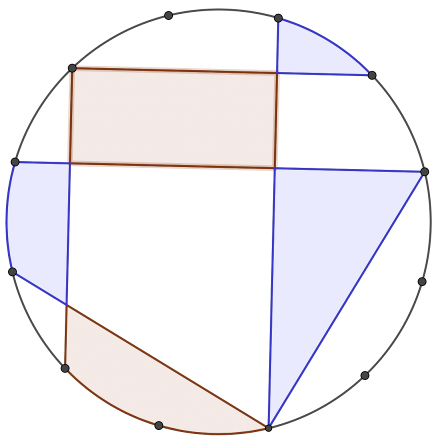
For example, $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$.
Anders took a positive integer x, and calculated that $x^{15} = 4\ 747\ 561\ 509\ 943$. What is the value of x?

Problem 7. In the figure below, two squares with parallel sides are drawn (the ones with black sides). The big square has side length 3 and the small has side length 2. What is the area of blue shaded quadrilateral?



Problem 8. The students in the UVS-village live in 49 skyscrapers which are evenly spaced along the main street. One student lives in the first house, two students live in the second house, and so on, until house 49 where 49 students live. When it's time to organise a big maths competition, the organisers want to know where to host it in order to minimise the total travel distance for all the students. Which house should they choose?

Problem 9. Prove that the sum of the blue areas is equal to the sum of the red areas. The figure is a circle and the points are evenly spaced.



Problem 10. The numbers $1, 2, 3, ..., 1000$ are written on the board. Kevin picks 12 of them, and erases the rest. He then notes that no sum of some numbers left on the board is a perfect power. Is this possible?
Note: A perfect power is a number of the form n^k for integers n and $k \geq 2$. For example, $27 = 3 \cdot 3 \cdot 3 = 3^3$, $49 = 7 \cdot 7 = 7^2$, $128 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$ are perfect powers.